OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4728

Mechanics 1

Tuesday **10 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

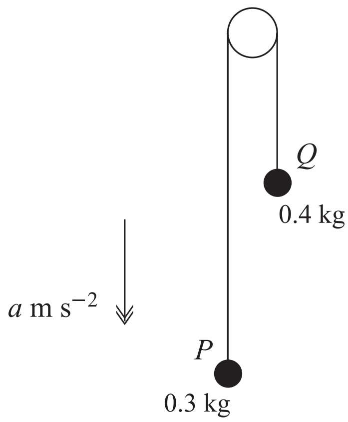
INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages and 3 blank pages.

[Turn over

1



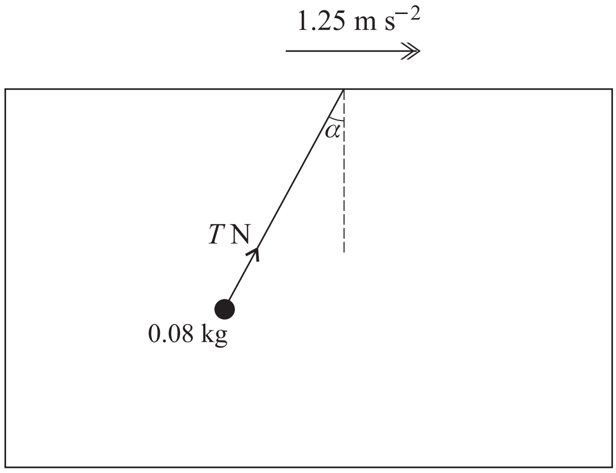
Particles P and Q, of masses 0.3 kg and 0.4 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is in motion with the string taut and with each of the particles moving vertically. The downward acceleration of P is a m s^{-2} (see diagram).

(i) Show that $a = -1.4$. [4]

Initially P and Q are at the same horizontal level. P's initial velocity is vertically downwards and has magnitude 2.8 m s^{-1}.

(ii) Assuming that P does not reach the floor and that Q does not reach the pulley, find the time taken for P to return to its initial position. [3]

2



An object of mass 0.08 kg is attached to one end of a light inextensible string. The other end of the string is attached to the underside of the roof inside a furniture van. The van is moving horizontally with constant acceleration 1.25 m s^{-2}. The string makes a constant angle α with the downward vertical and the tension in the string is T N (see diagram).

(i) By applying Newton's second law horizontally to the object, find the value of $T \sin \alpha$. [2]

(ii) Find the value of T. [5]

3 A motorcyclist starts from rest at a point O and travels in a straight line. His velocity after t seconds is $v\,\mathrm{m\,s^{-1}}$, for $0 \leqslant t \leqslant T$, where $v = 7.2t - 0.45t^2$. The motorcyclist's acceleration is zero when $t = T$.

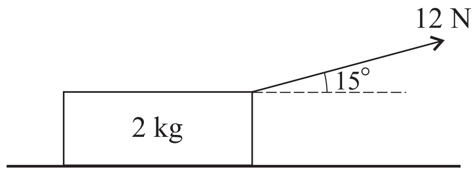
(i) Find the value of T. [4]

(ii) Show that $v = 28.8$ when $t = T$. [1]

For $t \geqslant T$ the motorcyclist travels in the same direction as before, but with constant speed $28.8\,\mathrm{m\,s^{-1}}$.

(iii) Find the displacement of the motorcyclist from O when $t = 31$. [6]

4



A block of mass 2 kg is at rest on a rough horizontal plane, acted on by a force of magnitude 12 N at an angle of 15° upwards from the horizontal (see diagram).

(i) Find the frictional component of the contact force exerted on the block by the plane. [2]

(ii) Show that the normal component of the contact force exerted on the block by the plane has magnitude 16.5 N, correct to 3 significant figures. [2]

It is given that the block is on the point of sliding.

(iii) Find the coefficient of friction between the block and the plane. [2]

The force of magnitude 12 N is now replaced by a horizontal force of magnitude 20 N. The block starts to move.

(iv) Find the acceleration of the block. [5]

5 A man drives a car on a horizontal straight road. At $t = 0$, where the time t is in seconds, the car runs out of petrol. At this instant the car is moving at $12\,\mathrm{m\,s^{-1}}$. The car decelerates uniformly, coming to rest when $t = 8$. The man then walks back along the road at $0.7\,\mathrm{m\,s^{-1}}$ until he reaches a petrol station a distance of 420 m from his car. After his arrival at the petrol station it takes him 250 s to obtain a can of petrol. He is then given a lift back to his car on a motorcycle. The motorcycle starts from rest and accelerates uniformly until its speed is $20\,\mathrm{m\,s^{-1}}$; it then decelerates uniformly, coming to rest at the stationary car at time $t = T$.

(i) Sketch the shape of the (t, v) graph for the man for $0 \leqslant t \leqslant T$. [Your sketch need not be drawn to scale; numerical values need not be shown.] [5]

(ii) Find the deceleration of the car for $0 < t < 8$. [2]

(iii) Find the value of T. [4]

6

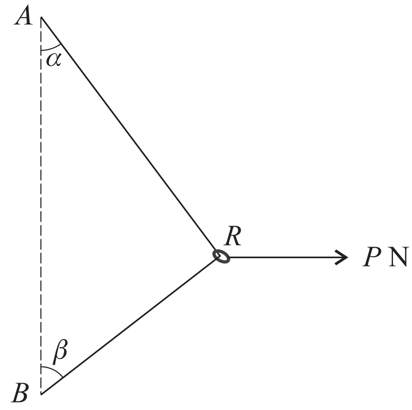


Fig. 1

A smooth ring R of weight W N is threaded on a light inextensible string. The ends of the string are attached to fixed points A and B, where A is vertically above B. A horizontal force of magnitude P N acts on R. The system is in equilibrium with the string taut; AR makes an angle α with the downward vertical and BR makes an angle β with the upward vertical (see Fig. 1).

(i) By considering the vertical components of the forces acting on R, show that $\alpha < \beta$. [3]

(ii)

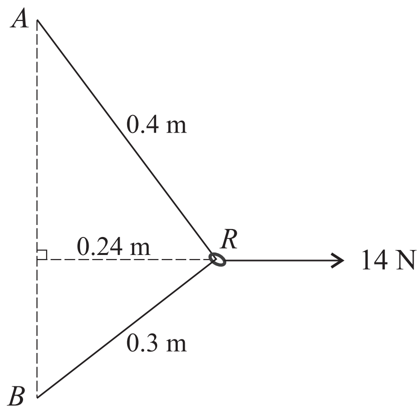


Fig. 2

It is given that when $P = 14$, $AR = 0.4$ m, $BR = 0.3$ m and the distance of R from the vertical line AB is 0.24 m (see Fig. 2). Find

(a) the tension in the string, [3]

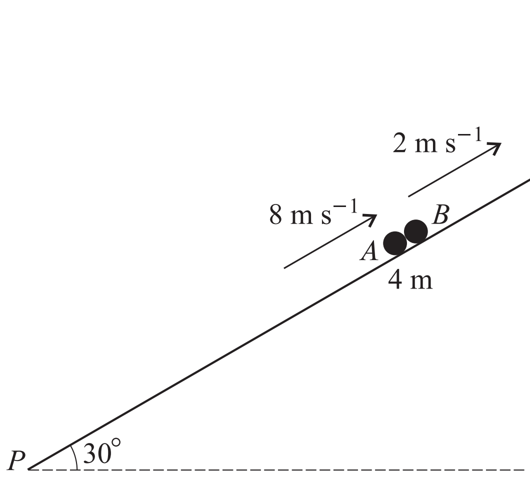
(b) the value of W. [3]

(iii) For the case when $P = 0$,

(a) describe the position of R, [1]

(b) state the tension in the string. [1]

7



PQ is a line of greatest slope, of length 4 m, on a smooth plane inclined at 30° to the horizontal. Particles A and B, of masses 0.15 kg and 0.5 kg respectively, move along PQ with A below B. The particles are both moving upwards, A with speed $8\,\mathrm{m\,s^{-1}}$ and B with speed $2\,\mathrm{m\,s^{-1}}$, when they collide at the mid-point of PQ (see diagram). Particle A is instantaneously at rest immediately after the collision.

(i) Show that B does not reach Q in the subsequent motion. [8]

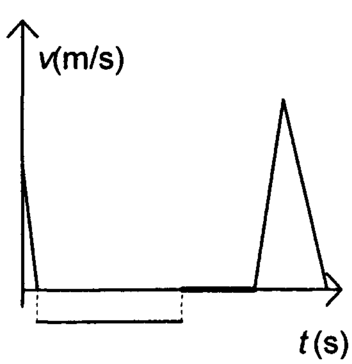
(ii) Find the time interval between the instant of A's arrival at P and the instant of B's arrival at P. [6]

1	(i)	$0.3g - T = 0.3a$ and $T - 0.4g = 0.4a$ $-0.1g = 0.7a$ $a = -1.4$ See appendix for substituting $a = -1.4$	M1 A1 M1 A1	 [4]	For using Newton's second law (either particle) condone 0.3ga,0.4ga and !(LHS) Both correct. ***SR*** Accept T -0.3g = 0.3a etc as correct only if consistent with a shown as upwards for P on c's diagram Eliminating T AG
	(ii)	$0 = 2.8t - ½\, 1.4t^2$ $0 = t(2.8 - 0.7t)$ Time taken is 4 s	M1 M1 A1	 [3]	For using $s = ut + ½\, at^2$ with s = 0 Solving QE From correct equation only
		OR $(0.3 + 0.4)a = (0.3 - 0.4)g$ $a = -1.4$	 M2 A1 A1	 [4]	For using $(m_1 + m_2)a = (m_1 - m_2)g$ No application of *SR* shown above AG
	(i)	$0 = 2.8 + -1.4t$ $t = 2.8/1.4$ Time taken is 4 s	M1 M1 A1	 [3]	For using v = u + at with v = 0 Solve for t, and double or any other complete method for return time
	(ii)				

2	(i)	$T\sin\alpha = 0.08 \times 1.25$ $= 0.1$	M1 A1	 [2]	Newton's second law condone cos, and 0.08g for mass but not part of force
	(ii)	$T\cos\alpha = 0.08g$ $T^2 = 0.1^2 + 0.784^2$ or $\alpha = 7.3°$ $T = 0.79$	M1 A1 M1 A1 A1	 [5]	Resolving forces vertically, condone sin May be implied by $T^2 = 0.1^2 + 0.784^2$ For eliminating α or T $\alpha = 7.3°$ or better Accept anything rounding to 0.79

3	(i)	 $a = 7.2 - 0.9t$ $T = 8$ See also special case in appendix.	M1 A1 M1 A1	 [4]	For using a = dv/dt For attempting to solve $a(t) = 0$
	(ii)	$v(T) = 28.8$ See also special case in appendix.	B1	[1]	AG (From $7.2 \times 8 - 0.45 \times 8^2$)
	(iii)	 $s = 3.6t^2 - 0.15t^3$ (+C) $s = 153.6$ (+C) s at constant speed = 662.4 Displacement is 816 m	M1 A1 DM1 A1 B1ft A1ft	 [6]	For using $s = \int v\,dt$ For finding s(T or 31) or using limits (0) to T or (0) to 31 (dep on integration) Condone +C For using (31 – cv T) ×28.8 cv 153.6 + cv 662.4 (non-zero numerical)

4	(i)	$F = 12\cos 15^\circ$ Frictional component is 11.6 N	M1 A1 [2]	Resolve horizontally (condone sin) Accept 12cos 15°
	(ii)	$N + 12\sin 15^\circ = 2g$ Normal component is 16.5 N	M1 A1 [2]	Resolve vert 3 forces (accept cos) AG
	(iii)	$11.591... = \mu\ 16.494...$ Coefficient is 0.7(0)	M1 A1ft [2]	For using cv $F = \mu$ cv N Ft cv F to 2 sf. $\mu = 0.7027....$
	(iv)	$N = 2g$ $F = 19.6 \times 0.7027...$ $20 - 13.773... = 2a$ Acceleration is 3.11 ms^{-2} ***MISREAD*** (omits "horizontal") $N = 2g - 20\sin 15$ $F = 0.7027 \times 14.4$ $20\cos 15 - 10.14 = 2a$ Acceleration is 4.59 ms^{-2}	B1 M1 M1 A1ft A1 [5] MR-1 B1ft M1 M1 A1ft A1ft [4]	 For using Newton's second law cv Tractive - cv Friction (e.g. from (i)) Accept either 3.11 or 3.12 only All A and B marks now ft. Subtract "MR-1" from initial B1 or final A1 (not A1ft in main scheme). Equals 14.42... Equals 10.1.... For using Newton's second law cv Tractive - cv Friction Accept 4.59, 4.6(0)

5	(i)		Graph with 5 straight line segments and with v single valued. Line segment for car stage Line segment for walk stage Line segment for wait stage 2 line segments for motor-cycle stage	B1 B1 B1 B1 B1 [5]	'Wait' line segment may not be distinguishable from part of the t axis. Attempt at all lines segments fully straight. Mainly straight, ends on t-axis Horizontal below t-axis. Ignore linking to axis. Can be implied by gap between walk and motor-cycle stages Inverted V not U, mainly straight. Condone vertex below x intercept.
	(ii)	$d = 12/8$ Deceleration is 1.5 ms^{-2}		M1 A1 [2]	Using gradient represents accn Or $a = -1.5\ ms^{-2}$
	(iii)	 $t_{walk} = 420/0.7$ $t_{motorcycle} = 42$ $T = 8 + 600 + 250 + 42 = 900$		M1 B1 B1 A1 [4]	Using area represents displacement. Accept 600 Ignore method

6	(i)	$T_A\cos\alpha - T_B\cos\beta = W$	M1	For resolving 3 forces vertically, condone Wg, sin
		$T_A = T_B$ (= T)	B1	May be implied or shown in diagram
		$\cos\alpha > \cos\beta \Rightarrow \alpha < \beta$	A1 [3]	AG
	(ii)(a)	$T\sin\alpha + T\sin\beta = 14$	M1	Resolve 3 forces horiz accept cos
		$\sin\alpha = 0.6$ and $\sin\beta = 0.8$	DM1	
		Tension is 10 N	A1 [3]	
	(ii)(b)	$10\cos\alpha - 10\cos\beta = W$	M1	Must use cv T, and W (not Wg)
		$\alpha = 36.9°$, $\beta = 53.1°$	DM1	Or $\cos\alpha = 0.8$ and $\cos\beta = 0.6$ ***SR*** -1 for assuming $\alpha + \beta = 90°$
		$W = 2$ See appendix for solution based on resolving along *RA* and *RB*.	A1 ft [3]	ft for $T/5$ (accept 1.99)
	(iii)	*R* is below *B*	B1	Accept *R* more than 0.5 m below *A*
		Tension is 1 N	B1 ft [2]	ft for $W/2$ accept $W/2$

7	(i)	Initial momentum $= 0.15\times 8 + 0.5\times 2$	B1		(or loss in A's momentum = 0.15×8 B1
		Final momentum = $0.5v$	B1		and gain in B's momentum = $0.5(v-2)$ B1)
		$0.15\times 8 + 0.5\times 2 = 0.5v$ (or $0.15\times 8 = 0.5\times(v - 2)$)	M1		For using the principle of conservation of momentum
		$v = 4.4$	A1	[4]	condone inclusion of g in all terms
		$(m)g\sin\alpha = (\pm)(m)a$	M1		***SR*** Awarded even if g in all terms
		$a = (\pm)4.9$	A1		Condone cos
		EITHER (see also part (ii))			
		$0 = 4.4^2 - 2\times 4.9s$	M1		For using $v^2 = u^2 + 2as$ with $v = 0$
		s = 1.97 or 1.98 m	A1ft		Accept $s < 2$ iff $s = 4.4^2/(2\times 4.9)$
		OR			
		$v^2 = 4.4^2 - 2\times 4.9\times 2$	M1		For using $v^2 = u^2 + 2as$ with $s = 2$
		$v^2 = -0.24$	A1ft		Accept $v^2 < 0$
		OR (see also part (ii))			
		$t = 4.4/4.9$ (=0.898) with either $s = 4.4\times 0.898 - 0.5\times 4.9\times 0.898^2$ or $s = (4.4 + 0)/2\times 0.898$	M1		Both parts of method needed
		s = 1.97 or 1.98 m	A1ft	[4]	Accept $s < 2$
	(ii)	$2 = ½4.9\, t_A^2$	M1		cv for acceleration
		$t_A = 0.904$	A1		Accept 0.903=<time=<0.904
		EITHER			
		$2 = (-4.4)t_B + ½\, 4.9t_B^2$	M1		Appropriate use of $s = ut + ½ at^2$
		$t_B = (4.4!\oplus(4.4^2 + 4\times 2.45\times 2))/4.9$	M1		Correct method for solving QE
			A1		
		$t_B = 2.17$	A1		2.171...
		$t_B - t_A = (2.17 - 0.9) = 1.27$ s			
		OR	M1		
		$t_{up} = 4.4/4.9$ (=0.898)	M1		Or using s_{up} to find t_{up}
		$(2 + 1.98) = 0.5\times 4.9\times t_{down}^2$	A1		$s = ut + ½ at^2$ with cv s in part (i)
		$t_{down} = 1.27$	A1		
		$t_B - t_A$ =(0.9 + 1.27 - 0.9) =1.27s			<u>Not the final answer</u>
		OR			
		$0 = 4.4t - ½\, 4.9t^2$ (i.e. approx 1.8 s to return to start)	M1		$s = ut + ½ at^2$ with $s = 0$ = 1.796
		$2 = 4.4t + 4.9t^2$	M1		
			A1		
		$t = 0.376$	A1	[5]	
		$t_B - t_A$ = 1.796+0.376-0.9 = 1.27s			